

L'Hospital's Rule

22. Using L'Hospital's rule, calculate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos ax}{1 - \cos bx}.$$

23. Calculate

$$\lim_{x \downarrow 1} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right).$$

24. Calculate

$$\lim_{x \rightarrow 0} \frac{x^2 + e^{x^3} - 1}{\ln(1+x) - \sin x}.$$

25. Calculate

$$\lim_{n \rightarrow \infty} 4^n \cdot \left(1 - \cos \frac{\theta}{2^n} \right).$$

Convexity and Concavity of Functions

26. Determine the domain, sign, and intervals of convexity and concavity for the function $y = xe^{\frac{1}{x}}$.

27. The function $f(x) = xe^{\frac{1}{x-2}}$ is given. Determine its domain, sign, and local extrema, and find intervals of convexity and concavity. Explain the geometric meaning of the limit

$$\lim_{x \downarrow 2} f(x),$$

and calculate it.

28. Find the intervals where the function

$$f(x) = \frac{x^2 + 1}{(x-1)^2}$$

is strictly increasing, and find the intervals of convexity and concavity. Explain the geometric meaning of the following limits:

$$\lim_{x \rightarrow \infty} f(x), \quad \lim_{x \rightarrow -\infty} f(x), \quad \lim_{x \uparrow 1} f(x), \quad \lim_{x \downarrow 1} f(x),$$

and calculate them.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.